Test questions **Definition of derivative, difference quotients.**

Calculators allowed with AP Calculus exam rules.

1. Given that $f'(x) = \lim_{h \to 0} \frac{4(\pi + h)^2 - 4\pi^2}{h}$

    a. $f(x)$ is what function?

    b. At what value of *x* is the derivative being calculated?

    c. Calculate the value of the limit. Show the work that leads to your answer.

2.

Distance *x* (cm)	0	3	4	6	9	10
Temperature $T(x)$ (^{o}C)	110	94	85	79	73	68

A metal wire of length 10 cm is heated at one end. The table above gives selected values of the temperature, $T(x)$ in degrees Celsius (^{o}C) of the wire *x* cm from the heated end. The function *T* is differentiable.

a. What theorem assures you that there a distance at which the temperature of the wire is 80^{o}C?

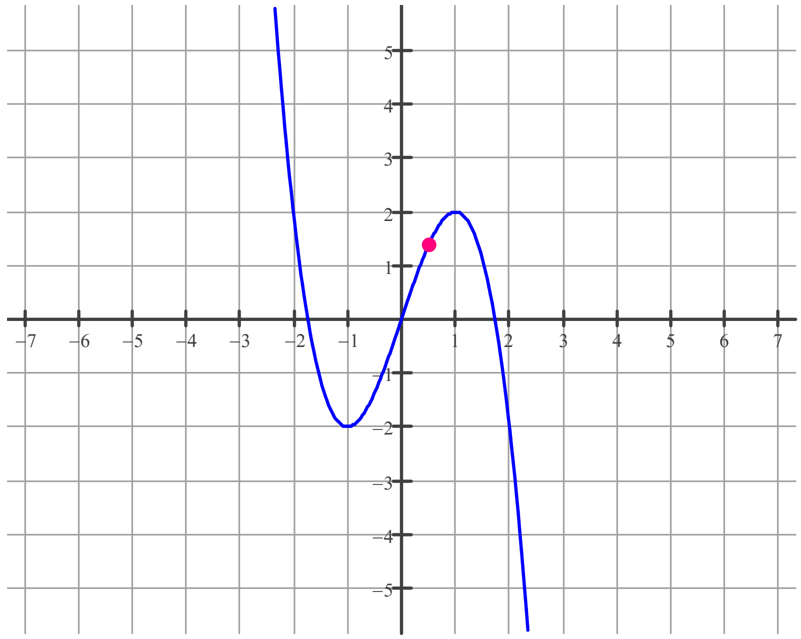
b. One of the hypotheses of the theorem from part a is that the function is continuous. What tells you that this function is continuous?

c. Find the average rate of change of *T* on the interval $6 \le x \le 10$. Include units of measure.

d. Find the approximate value of $T'(5)$. Include units of measure. Show the work that leads to your answer.

e. Given that $T'(3) = -6$, write an equation of the line tangent to the graph of T at $x = 3$ and use it to approximate $T(3.2)$.

3. The figure below shows the graph of a function, $g(x)$, with the point (0.5, 1.375) marked.



a. Sketch the tangent line at the point marked and use it to estimate $g'(0.5)$.

b. At what points does it appear that $g'(x)=0$?

c. On what open interval does it appear that $g'(x)>0$?

4. Use the definition of derivative to calculate $f'(x)$ for the function $f(x)=x^2+3x-7$. Show your work.

5. How is the average rate of change of a function over an interval related to the derivative of the function at a point in the interval? Are there any circumstances when this relationship may not be valid?

6. A function, f is continuous and its derivative is continuous between the points A(2,4) and B (5,10). Also $f'(2)=3$ and $f'(5)=-1$.

a. Sketch a graph of a function that meets these conditions.

b. Calculate m, the slope of the $\overline{AB}$.

c. Must there be a point $(c, f(c))$ on the graph of f between A and B where $f'(c)=m$? (Yes or no) Explain your reasoning.

Answers:

1. a. **[1 point]** $f\left(x\right)=4x^2$; b**. [1]** π ; c. **[2]** $8\pi \approx 25.133$

2. a. **[1]** Intermediate Value Theorem; b.**[1]** The function is given as differentiable and differentiability implies continuity. c. **[2]** –2.75 °C/cm. d. **[2]** –3 °C/cm
   e. **[3]** $y=94-6\left(x-3\right);T\left(3.2\right)\approx y\left(3.2\right)=92.8$ °C

3. a. **[2]** The actual slope is 2.25, estimates near 2 are reasonable. b**. [2]** (–1, –2) and (1,2)
   c. **[1]** (–1,1)

4. **[4]** $f'\left(x\right)=\lim\limits_{h\to 0}\dfrac{\left(x+h\right)^2+3(x+h)-7-\left(x^2+3x-7\right)}{h}$
   $$=\lim_{h\to 0}\frac{x^2+2xh+h^2+3x+3h-7-x^2-3x+7}{h}$$
   $$=\lim_{h\to 0}\frac{2xh+h^2+3h}{h}=\lim_{h\to 0}\left(2x+h+3\right)=2x+3$$

5. **[3]** The average rate of change is approximately the same as the value of the derivative at points in the interval. The shorter the interval, the better the approximation. This relationship may not be true if the function is not differentiable on the interval.

6. a. **[2]** sketch, b. **[1]** m = 2, c. **[2]** Yes. Since the derivative is continuous on the interval [2,5] and the derivative goes from 3 to –1, by the intermediate value theorem it must take on all the values between 3 and –1 including m = 2. **OR** by the Mean Value Theorem there must be a point where the derivative has the same value as the slope of the segment between the endpoints (= 2).